

Average

An average or an arithmetic mean of given data is the sum of the given observations divided by number of observations.

Important Formulae Related to Average of numbers

1. Average of first n natural number = $(n+1)/2$
2. Average of first n even number = $(n+1)$
3. Average of first n odd number = n
4. Average of consecutive number = $(\text{First number} + \text{Last number})/2$
5. Average of 1 to n odd numbers = $(\text{Last odd number} + 1)/2$
6. Average of 1 to n even numbers = $(\text{Last even number} + 2)/2$
7. Average of squares of first n natural numbers = $[(n+1)(2n+1)]/6$
8. Average of the cubes of first n natural number = $[n(n+1)^2]/4$
9. Average of n multiples of any number = $[\text{Number} * (n+1)]/2$

Concept 1

If the average of n_1 observations is a_1 ; the average of n_2 observations is a_2 and so on, then

Average of all the observations = $(n_1 * a_1 + n_2 * a_2 + \dots) / (n_1 + n_2 + \dots)$

Concept 2

If the average of m observations is a and the average of n observations taken out of is b , then

Average of rest of the observations = $(ma - nb) / (m - n)$

Example 1 : A man bought 20 cows in RS. 200000. If the average cost of 12 cows is Rs. 12500, then what will be the average cost of remaining cows?

Here $m = 20$, $n = 12$, $a = 10000$, $b = 12500$

average cost of remaining cows (20-12) cows = $(20*10000 - 12*12500)/(20-12) = \text{Rs } 6250$

Concept 3

If the average of n students in a class is a , where average of passed students is x and average of failed students is y , then

Number of students passed = $\frac{\text{Total Students} (\text{Total average} - \text{Average of failed students})}{(\text{Average of passed students} - \text{Average of failed students})}$
 $= \frac{n(a-y)}{(x-y)}$

Example 2: In a class, there are 75 students and their average marks in the annual examination is 35. If the average marks of passed students is 55 and average marks of failed students is 30, then find out the number of students who failed.

Here, $n = 75$, $a = 35$, $x = 55$, $y = 30$

Number of students who passed = $75(35 - 30)/(55 - 30) = 15$

Number of students who failed = $75 - 15 = 60$

Concept 4

If the average of total components in a group is a , where average of n components (1st part) is b and average of remaining components (2nd part) is c , then Number of remaining components (2nd part) = $\frac{n(a-b)}{(c-a)}$

Example 3 : The average salary of the entire staff in an office is Rs. 200 per day. The average salary of officers is Rs. 550 and that of non-officers is Rs. 120. If the number of officers is 16, then find the numbers of non-officers in the office.

Here $n = 16$, $a = 200$, $b = 550$, $c = 120$

Number of non-officer = $16(200 - 550)/(120 - 200) = 70$

Average Speed

Average speed is defined as total distance travelled divided by total time taken.

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Case 1

If a person covers a certain distance at a speed of A km/h and again covers the same distance at a speed of B km/h, then the average speed during the whole journey

will be

$\frac{2AB}{A+B}$

Case II

If a person covers three equal distances at the speed of A km/h, B km/h and C km/h respectively, then the average speed during the whole Journey will be

$\frac{3ABC}{AB+BC+CA}$

Case III

If distance P is covered with speed x, distance Q is covered with speed y and distance R is covered with speed z, then for the whole journey,
Average speed = $(P+Q+R+\dots)/(P/x+Q/y+R/z+\dots)$

Example 4 : A person covers 20 km distance with a speed of 5 km/h, then he covers the next 15 km with a speed of 3 km/h and the last 10 km is covered by him with a speed of 2 km/h. Find out his average speed for the whole journey.

$$\text{Average speed} = (20 + 15 + 10)/(20/5 + 15/3 + 10/2) = 3(3/14)$$

Case IV

If a person covers P part of his total distance with speed of x, Q part of total distance with speed of y and R part of total distance with speed of z, then
Average speed = $1/(P/x+Q/y+R/z+\dots)$